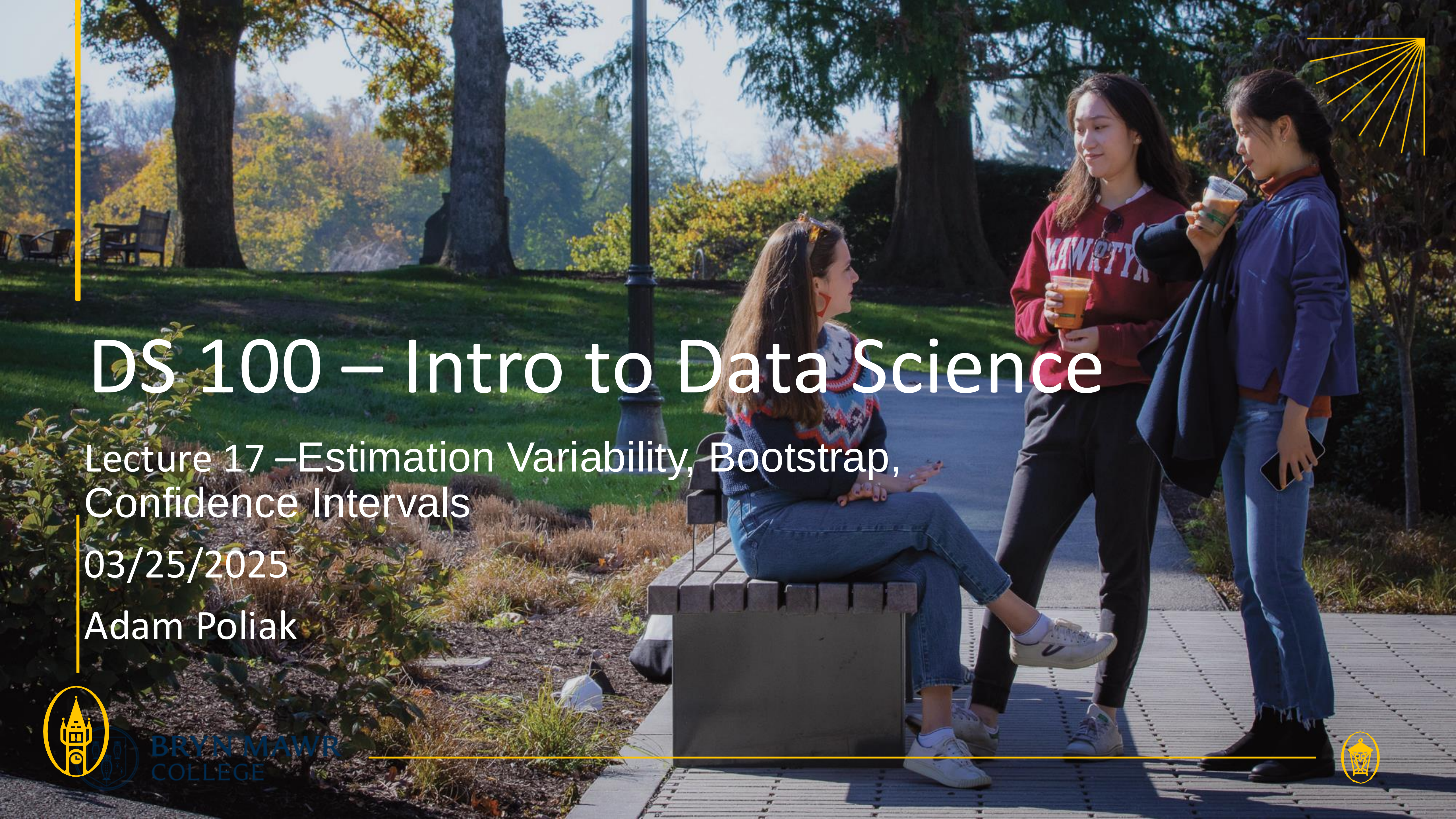

DS 100 – Intro to Data Science

Lecture 17 –Estimation Variability, Bootstrap, Confidence Intervals

03/25/2025

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BRYN MAWR COLLEGE

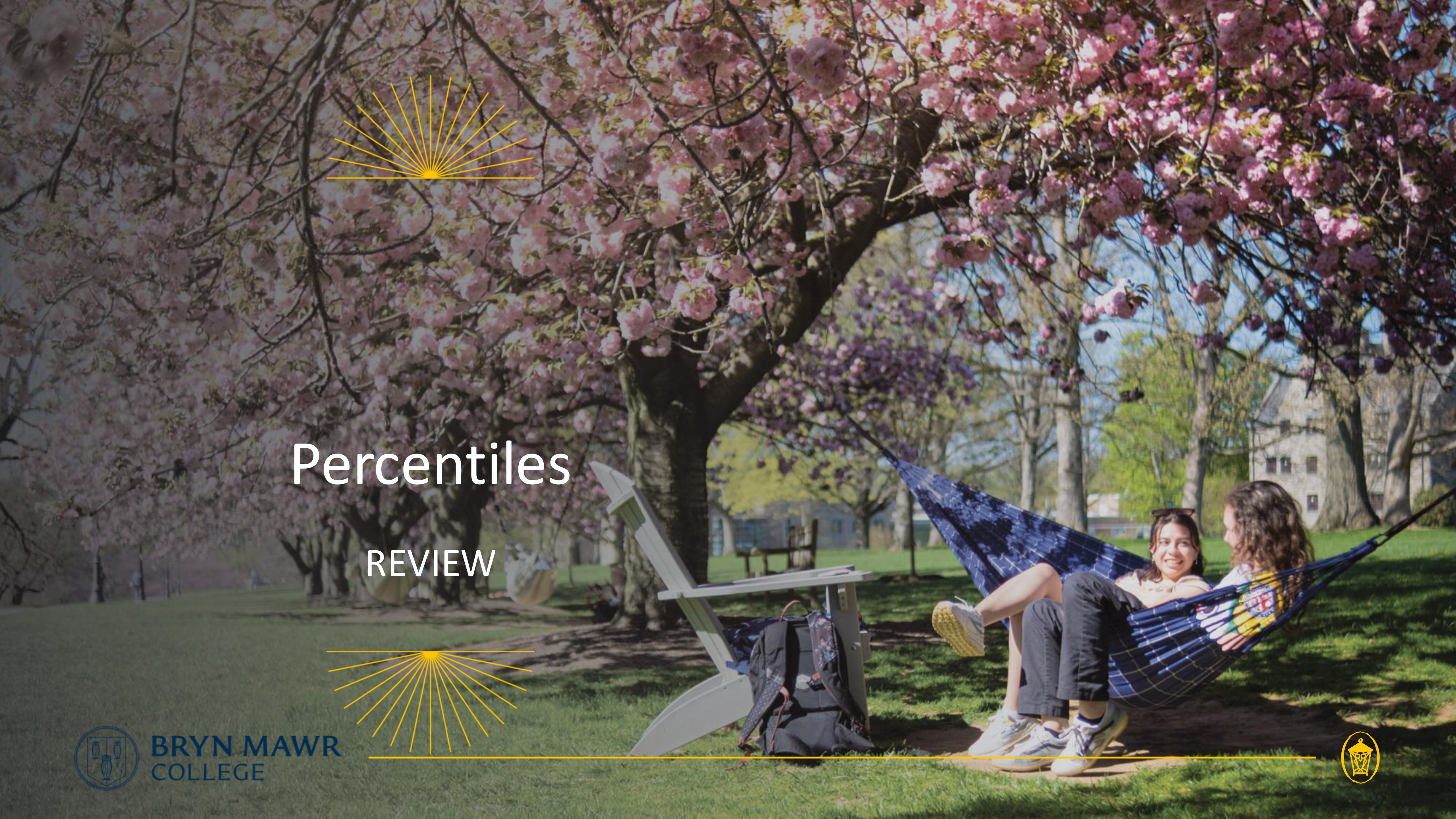
Announcements

Midsemester feedback form - https://forms.gle/M4jVdGTDgQknWAwo6

Midterm – Thursday (03/27)

No Lab today, No Homework this week

Project 2 (due Friday April 11th)



Percentiles

REVIEW

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The percentile Function

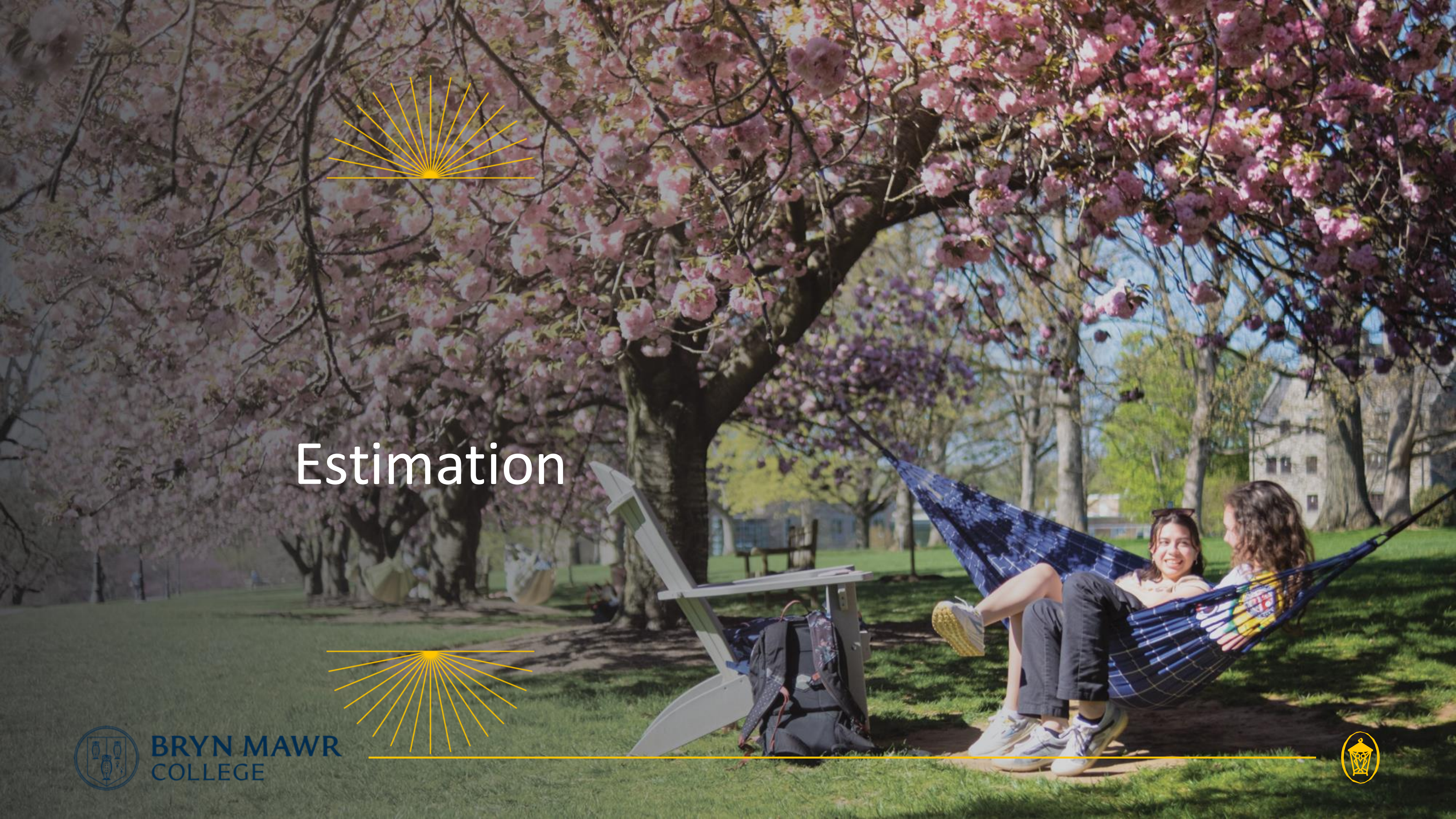
The *p*th percentile is the **smallest value** in a set that is **at least as large as *p*%** of the elements in the set

Function in the datascience module:

```
percentile(p, values)
```

p is between 0 and 100

Returns the *p*th percentile of the array



Estimation

Inference: Estimation

How do we calculate the value of an unknown parameter?

If you have a census (that is, the whole population):

- Just calculate the parameter and you’re done

If you don’t have a census:

- Take a random sample from the population
- Use a statistic as an **estimate** of the parameter

Estimation Variability

Variability of the Estimate

One sample → One estimate

But the random sample could have come out differently

And so the estimate could have been different

Big question:

- How different would it be if we estimated again?

Quantifying Uncertainty

The estimate is usually not exactly right.

Variability of the estimate tells us something about how accurate the estimate is:

Estimate = Parameter + Error

How accurate is the estimate, usually?
How big is a typical error?
When we have a census, we can do this by simulation

Where to Get Another Sample?

We want to understand errors of our estimate

Given the **population**, we could simulate

- ...but we only have the **sample**!

To get many values of the estimate, we needed many random samples

Can’t go back and sample again from the population:

- No time, no money

Stuck?

Boostrap

The Bootstrap

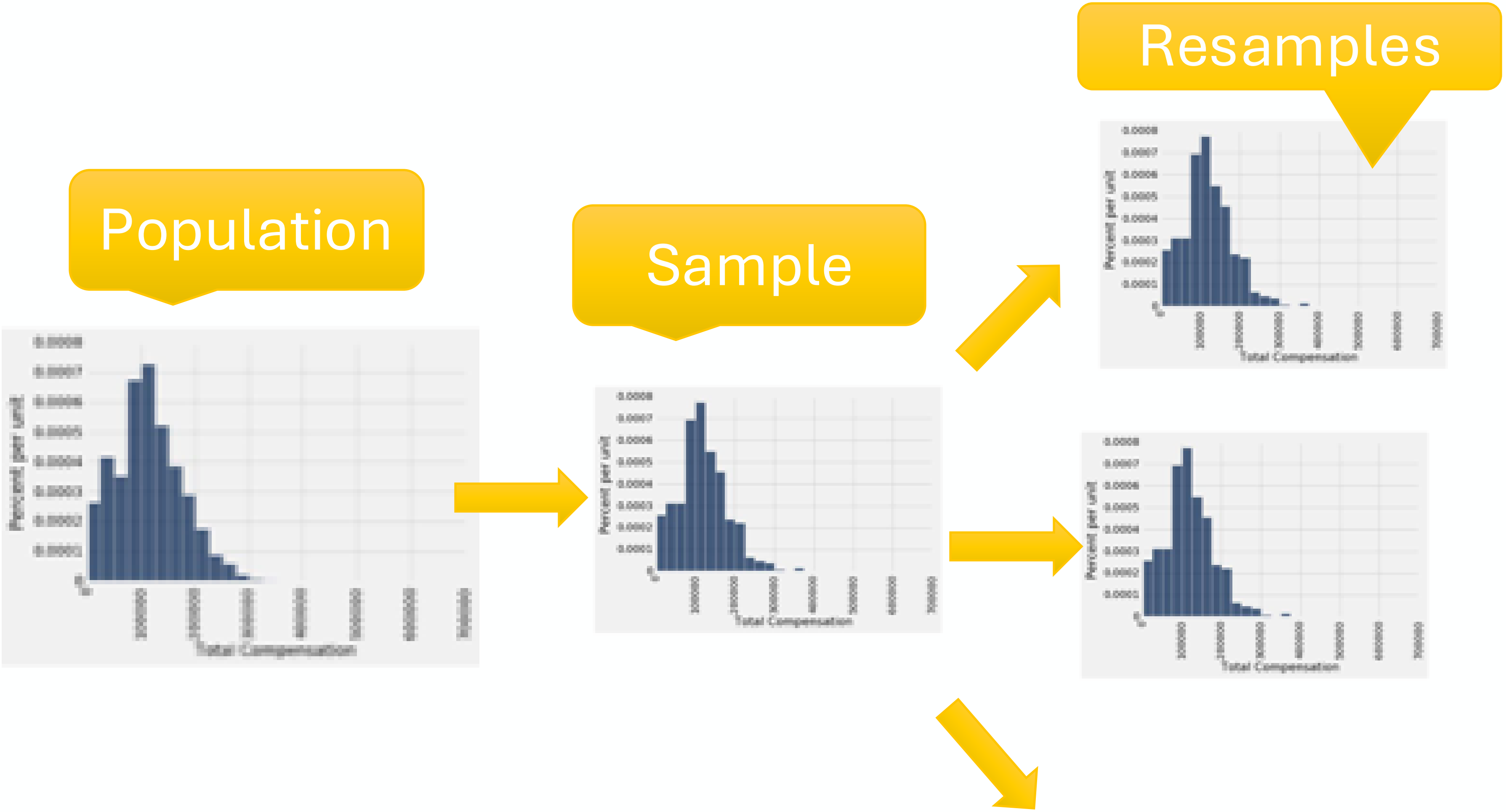
A technique for simulating repeated random sampling

All that we have is the original sample

- ... which is large and random
- Therefore, it probably resembles the population

So we sample at random from the original sample!

How the Bootstrap Works

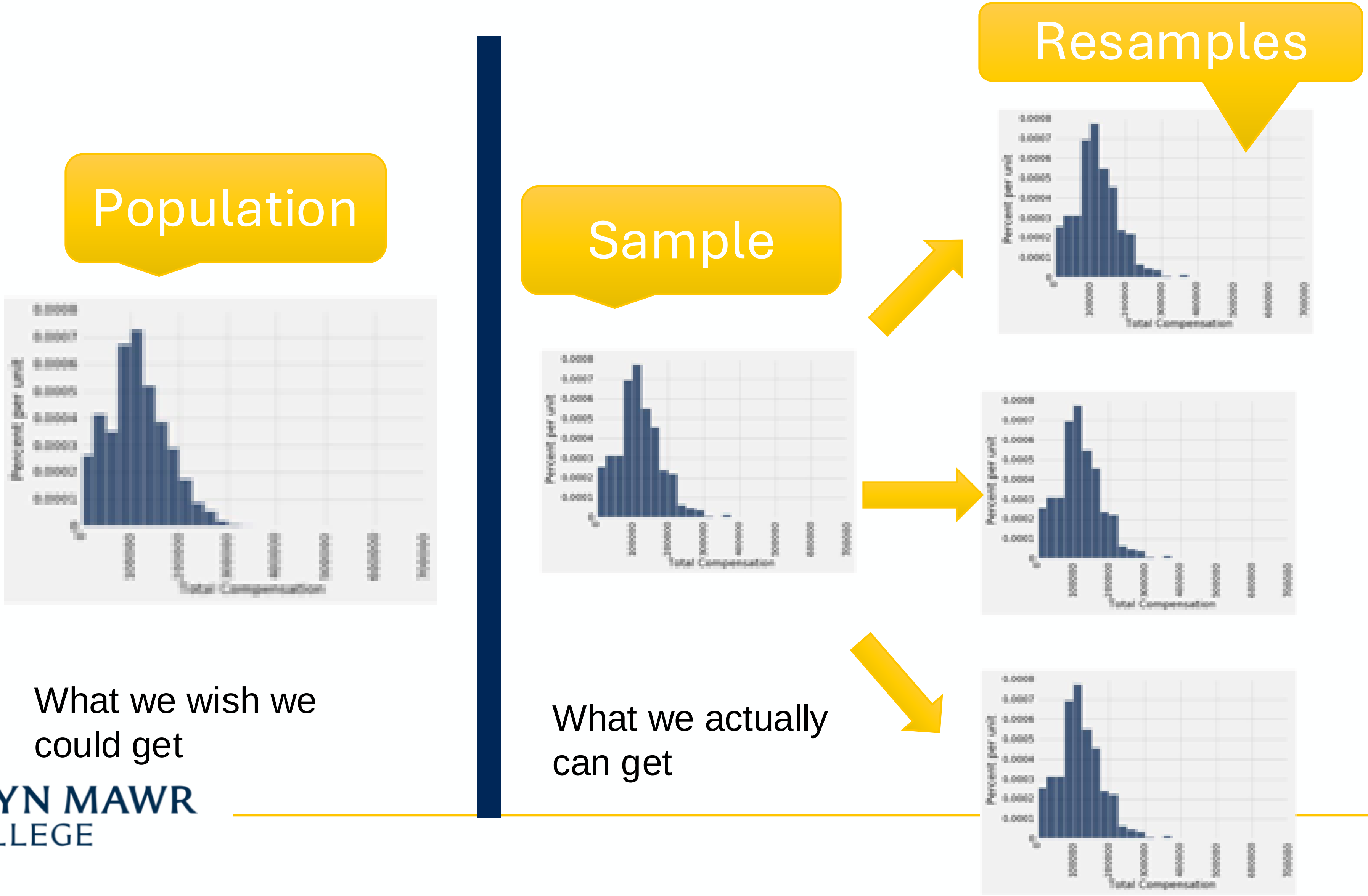


Population

Sample

Resamples

Why the Bootstrap Works



Population

Sample

Resamples

What we wish we could get

What we actually can get

Real World vs Bootstrap World

Real World

- True probability distribution (population)
    - Random sample 1
        - Estimate 1
    - Random sample 2
        - Estimate 2
    - …
    - Random sample 1000
        - Estimate 1000

Bootstrap World

- Empirical distribution of original sample ("population")
    - Bootstrap sample 1
        - Estimate 1
    - Bootstrap sample 2
        - Estimate 2
    - …
    - Bootstrap sample 1000
        - Estimate 1000

Hope: these two scenarios are analogous

The Bootstrap Principle

The bootstrap principle:

- **Bootstrap-world** sampling ≈ **Real-world** sampling

Not always true!

- ... but reasonable if sample is large enough

We hope that:

a) Variability of bootstrap estimate

b) Distribution of bootstrap errors

...are similar to what they are in the real world

Key to Resampling

From the original sample,

- draw at random
- with replacement
- as many values as the original sample contained

The size of the new sample has to be the same as the original one, so that the two estimates are comparable

Confidence Intervals

95% Confidence Interval

Interval of **estimates of a parameter**

Based on random sampling

95% is called the confidence level

- Could be any percent between 0 and 100
- Higher level means wider intervals

The **confidence is in the process** that gives the interval:

- It generates a “good” interval about 95% of the time

Use Methods Appropriately

Can You Use a CI Like This?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- About 95% of the mothers in the population were between 26.9 years and 27.6 years old.

Answer:

- **False.** We’re estimating that their **average age** is in this interval.

Is This What a CI Means?

An approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

There is a 0.95 probability that the average age of mothers in the population is in the range 26.9 to 27.6 years.

Answer:

False. The average age of the mothers in the population is unknown but it’s a constant. It’s not random. No chances involved

When *Not* to use the Bootstrap

if you’re trying to estimate very high or very low percentiles, or min and max

If you’re trying to estimate any parameter that’s greatly affected by rare elements of the population

If the probability distribution of your statistic is not roughly bell shaped (the shape of the empirical distribution will be a clue)

If the original sample is very small

Using a CI For Hypothesis Testing

Null hypothesis: **Population average = *x***

Alternative hypothesis: **Population average $\neq$ *x***

Cutoff for P-value: *p*%

Method:

- Construct a (100-*p*)% confidence interval for the population average
- If *x* is not in the interval, reject the null
- If *x* is in the interval, can't reject the null

Data Science in this course

Exploration

- Discover patterns in data
- Articulate insights (visualizations)

Inference

- Make reliable conclusions about the world
- Statistics is useful

Prediction

- **Informed guesses about unseen data**