DS 100 – Intro to Data Science

Lecture 15 – A/B Testing

03/18/2025

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Announcements

Midsemester feedback form - https://forms.gle/M4jVdGTDgQknWAwo6

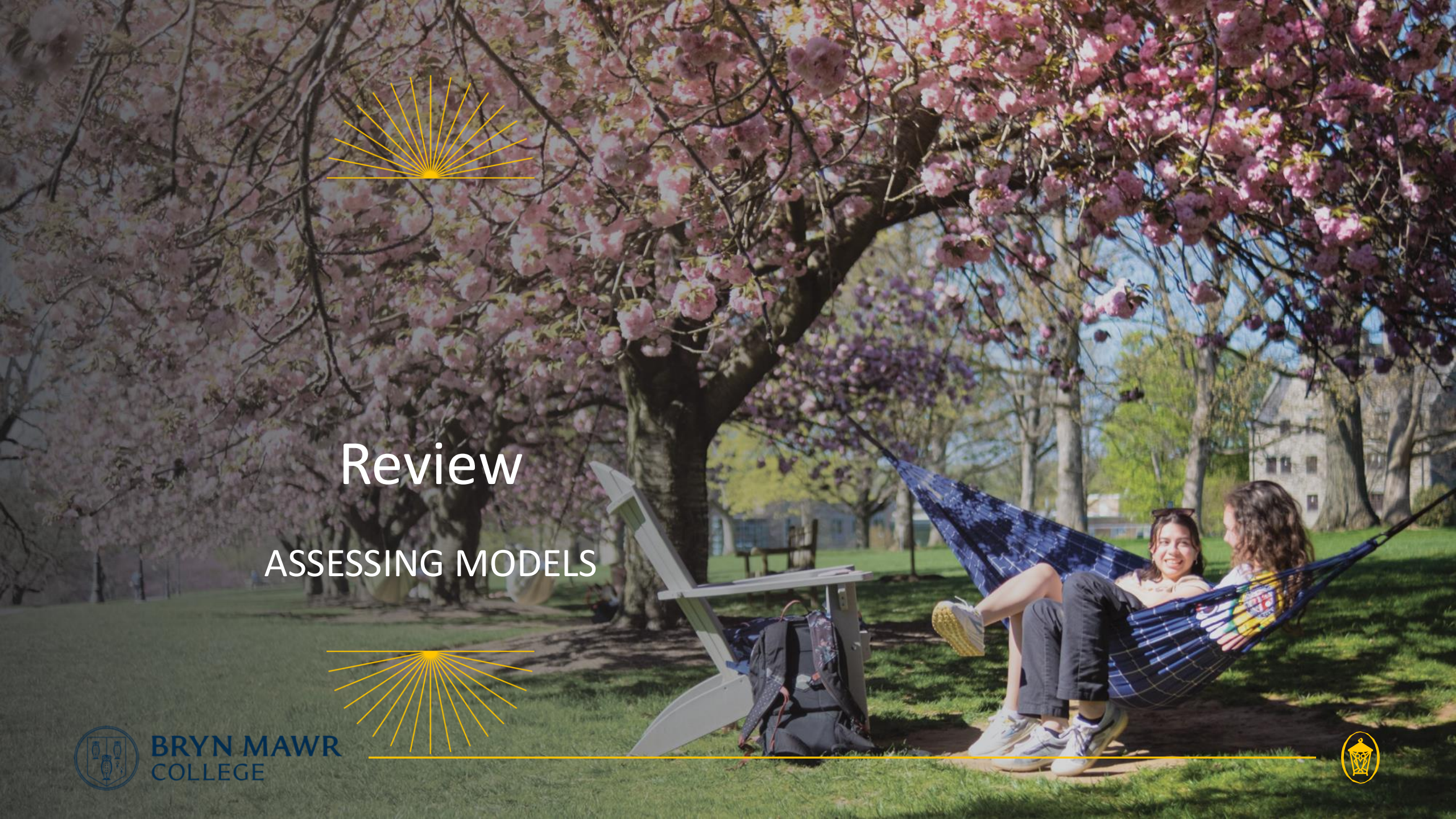
Midterm – Thursday (03/27)

Lab06 – Great British Bake-Off (A/B Testing)

- Due Friday (03/21)

HW06 – Hypothesis Testing

- Due Wednesday (03/19)



Review

ASSESSING MODELS

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Models

A model is a set of assumptions about the data

In data science, many models involve assumptions about processes that involve randomness:

- "Chance models"

Key question: does the model fit the data?

Approach to Assessing Models

If we can simulate data according to the assumptions of the model, we can learn what the model predicts

We can compare the model's predictions to the observed data

Here, "observed data" == what actually happened

If the data and the model's predictions are not consistent, that is evidence against the model

Steps in Assessing a Model

Choose a statistic to measure the “discrepancy” between the model and the data

Simulate statistic under the assumptions of the model

Compare the data to the model’s predictions:

- Draw a histogram of the simulated values
- Compute the observed statistic from the real sample

If the observed statistic is far from the histogram, that is evidence again the model

Comparing Distributions
A New Statistic

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Total Variation Distance

Every distance has a computational recipe

Total Variation Distance (TVD):

- For each category, compute the difference in proportions between two distributions
- Take the absolute value of each difference
- Sum, and then divide the sum by 2

Testing Hypotheses

A test chooses between two views of how data are generated

What are these views called?

- Answer: **hypotheses**

The test picks the hypothesis that is better supported by the observed data

What is the method for choosing between the hypotheses?

- Simulate data under one of the hypotheses
- Compare the simulation results and the observed data
- Pick one of the hypotheses based on whether the simulated results and observed data are consistent

Null and Alternative

The method only works if we can simulate data under one of the hypotheses.

Null hypothesis

- A well defined chance model about how the data were generated
- We can simulate data under the assumptions of this model
  - "Under the null hypothesis"

Alternative hypothesis:

- A different view about the origin of the data

Test Statistic

The statistic that we choose to simulate, to decide between the two hypotheses

Questions before choosing the statistic:

What values of the statistic will make us lean towards the null hypothesis?

What values will make us lean towards the alternative?

- Preferably, the answer should be just a “high” or just a “low” value
- Try to avoid “both high and low”

Prediction Under the Null Hypothesis

Simulate the test statistic under the null hypothesis

- Draw the histogram of simulated values
- **The empirical distribution of the statistic under the null hypothesis**

It is a prediction about the statistic, made by the null hypothesis

- It shows all the likely values of the statistic
- Also how likely they are (**if the null hypothesis is true)**

The probabilities are approximate, because we can't generate all the possible random samples

Conclusion of the Test

Resolve choice between null and alternative hypotheses

Compare the **observed test statistic** and its empirical distribution under the null hypothesis

If the observed value is not **consistent** with the empirical distribution

- The test favors the alternative
- “data is more consistent with the alternative”

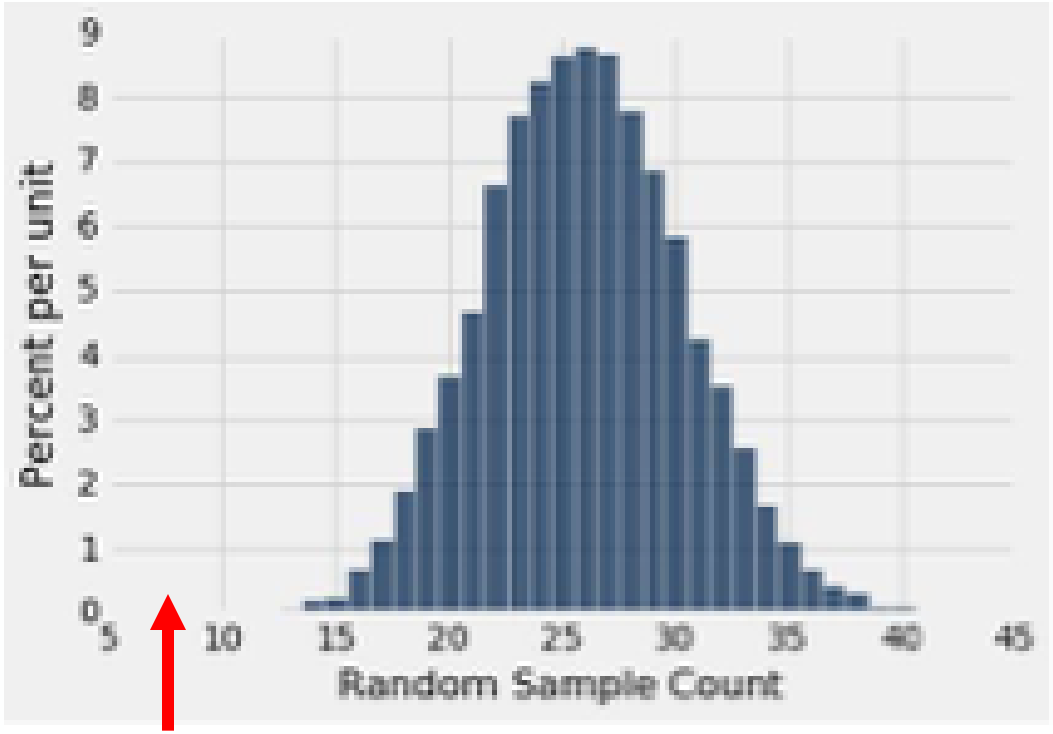
Whether a value is consistent with a distribution:

- A visualization may be sufficient
- If not, there are conventions about “consistency”

Statistical Significance

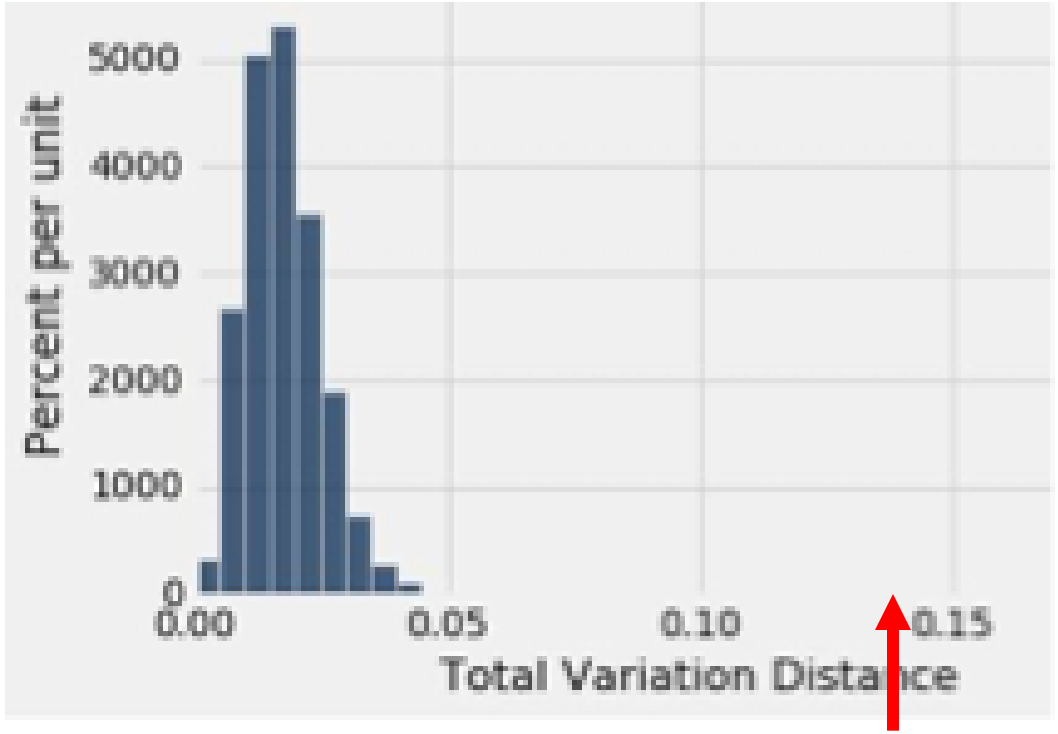
Tail Areas

Alabama Jury



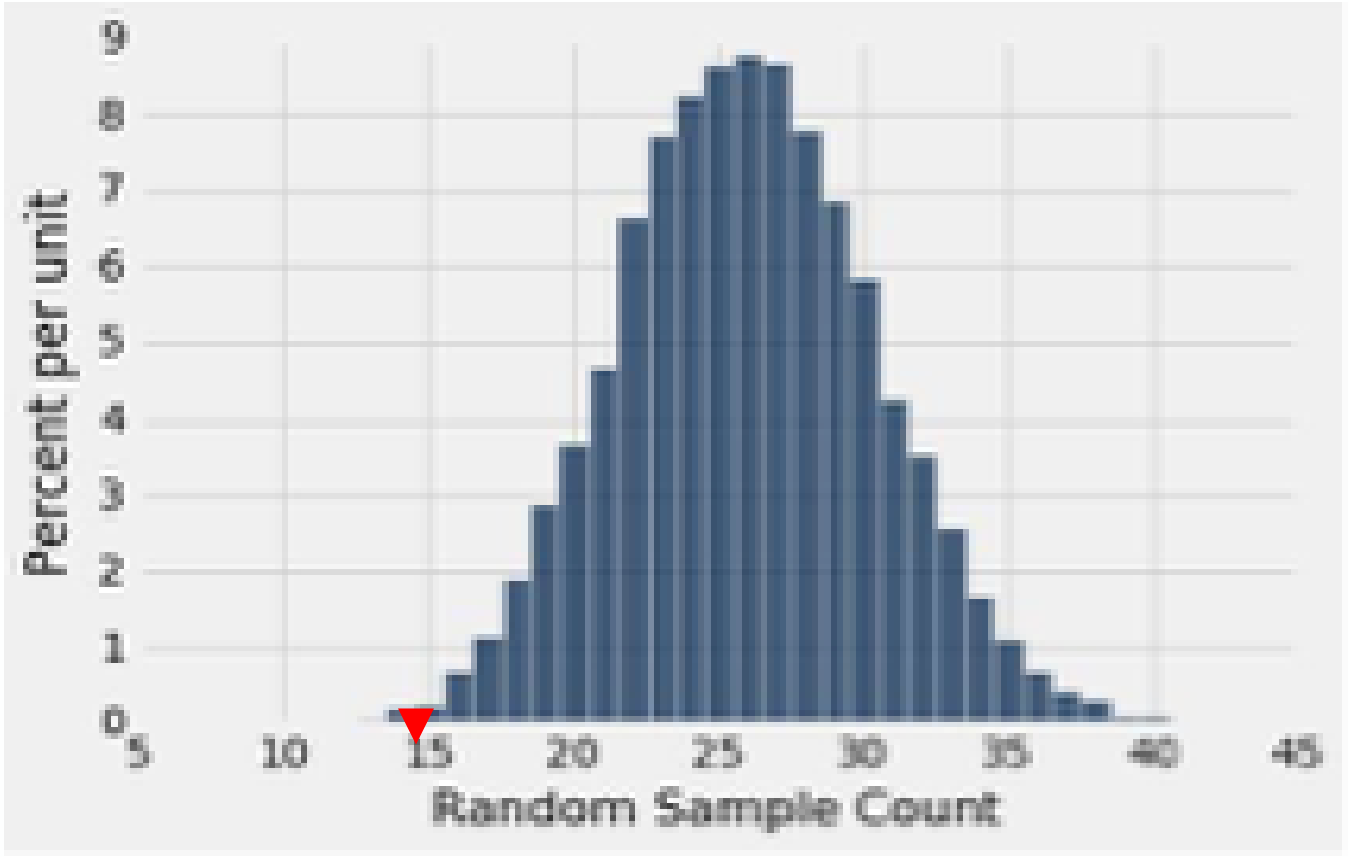
Observed Number (8)

Alameda Jury



Observed TVD (0.14)

Not so clear example

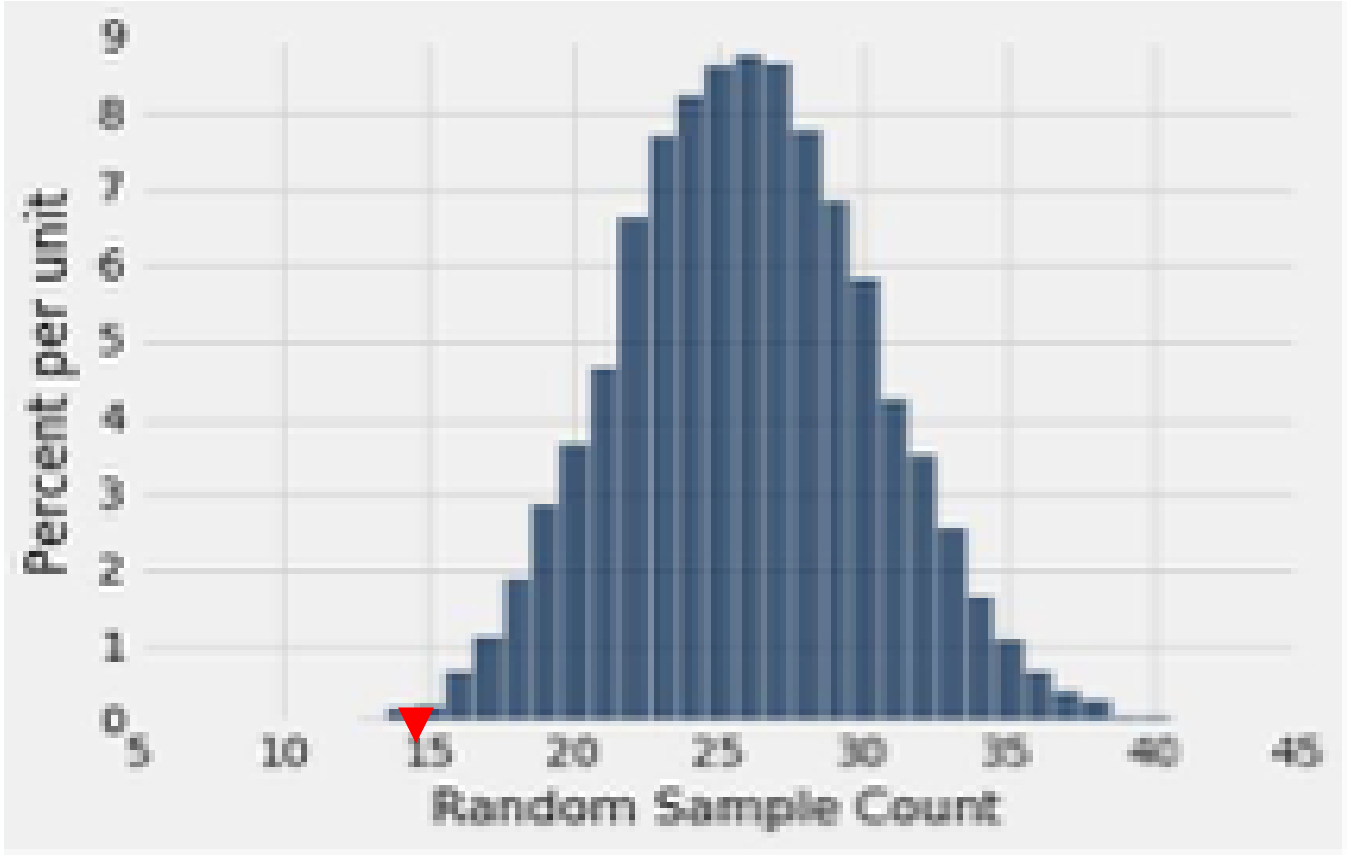


Observed Number (18)

Conventions About Inconsistency

“Inconsistent with the null”: The test statistic is in the tail of the empirical distribution under the null hypothesis

Is the test statistic in the tail of the empirical distribution under the null hypothesis?



Observed Number (18)

Conventions On Inconsistency

“Inconsistent with the null”: The test statistic is in the tail of the empirical distribution under the null hypothesis

“In the tail,” first convention:

- The area in the tail is less than 5%
- The result is “statistically significant”

”In the tail,” second convention:

- The area in the tail is less than 1%
- The result is “highly statistically significant”

Definition of the P-value

Formal name: **observed significance level**

The *P*-value is the chance,

- Under the null hypothesis,
- That the test statistic
- Is equal to the value that was observed in the data
- Or is even further in the direction of the tail

Example

Scenario: After the midterm, students in section 1 (of 27 students) noticed that their scores were on average lower than the rest of the class.

Question:

Why did the section do worse than others?

Potential Answers:

Null Hypothesis: The average score of the students in the lab is like the average score of the same number of students picked at random from the class

Alternative Hypothesis: No, the average is too low

Example

Scenario: After the midterm, students in section 1 (of 27 students) noticed that their scores were on average lower than the rest of the class.

Question:

Did the 27 students do lower by chance?

Potential Answers:

Null Hypothesis: The average score of the students in the lab is like the average score of the same number of students picked at random from the class

Alternative Hypothesis: No, the average is too low

Statistic to measure

The average score per section (27 students)

Assessing a Model

Choose a statistic to measure the “discrepancy” between model and data

- Average score per 27 students

Simulate the statistic under the model’s assumptions

- `np.average(scores_only.sample(27, with_replacement=False))`

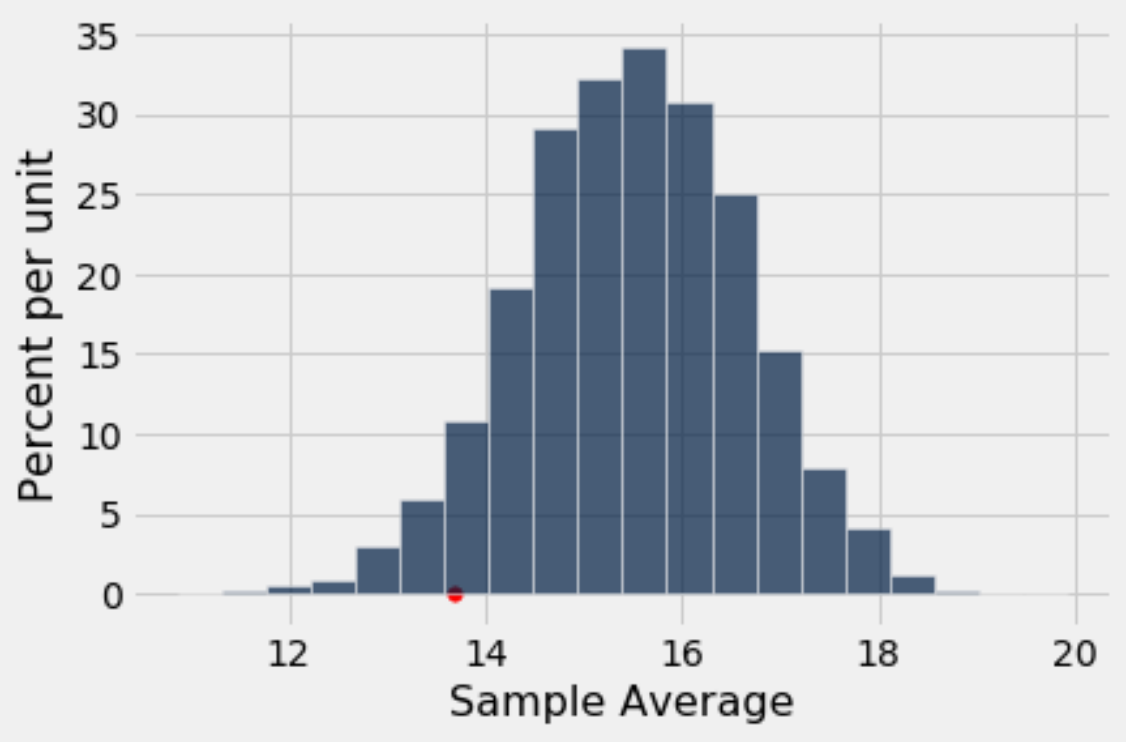
Compare the data to the model’s predictions:

- Draw a histogram of simulated values of the statistic
- Compute the observed statistic from the real sample

Compute the p-value

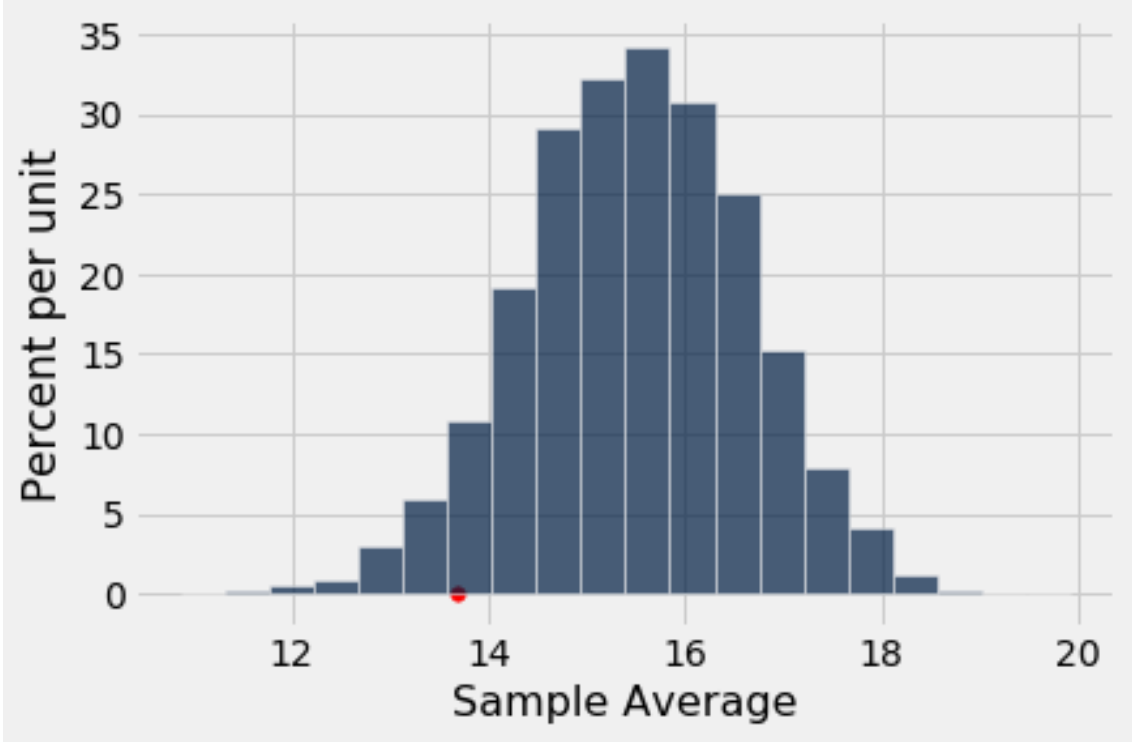
The *P*-value is the chance,

- Under the null hypothesis, that the test statistic, is equal to the value that was observed the data, or is even further in the direction of the tail



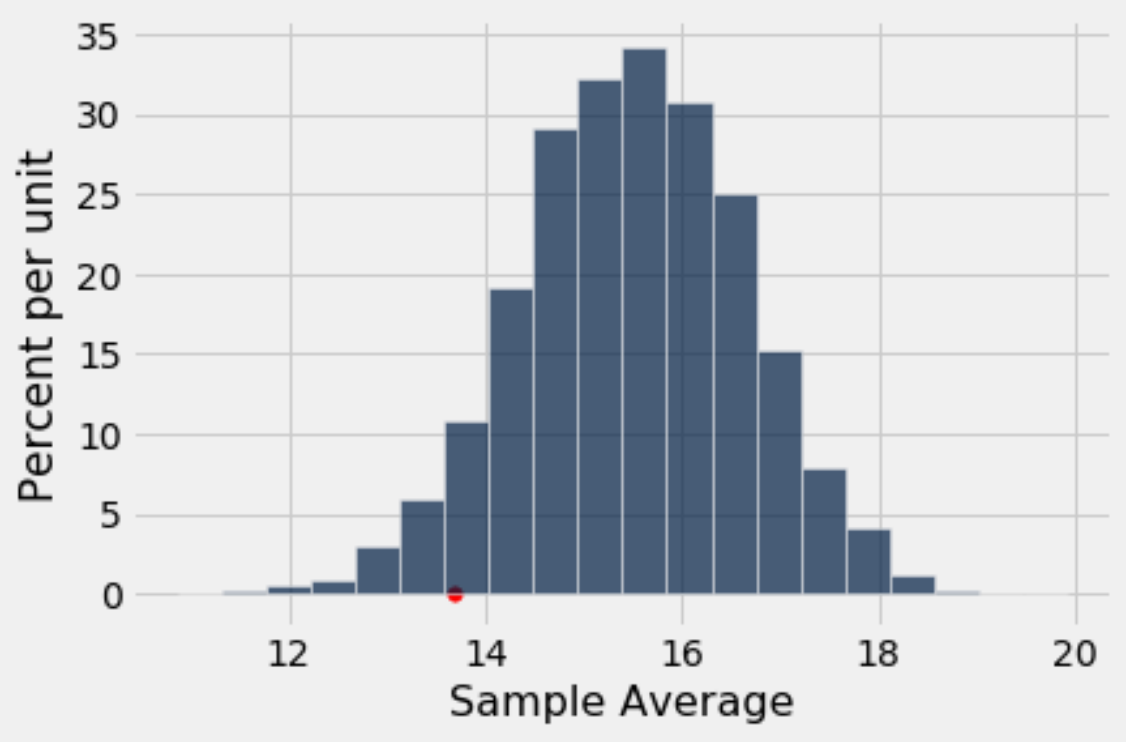
Compute the p-value

$$\text{Probability (A)} = \frac{number\ of\ outcomes\ that\ make\ A\ happen}{total\ number\ of\ outcomes}$$



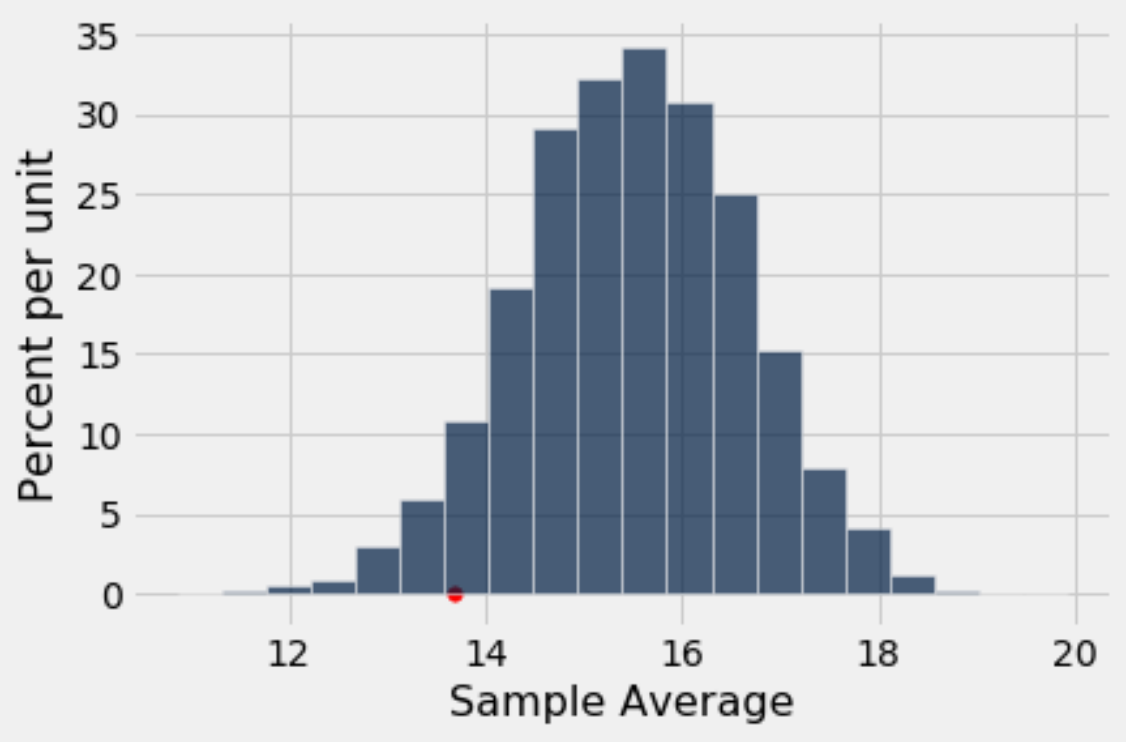
Compute the p-value

A = the sampled statistic was less than or equal to the observed statistic



Compute the p-value

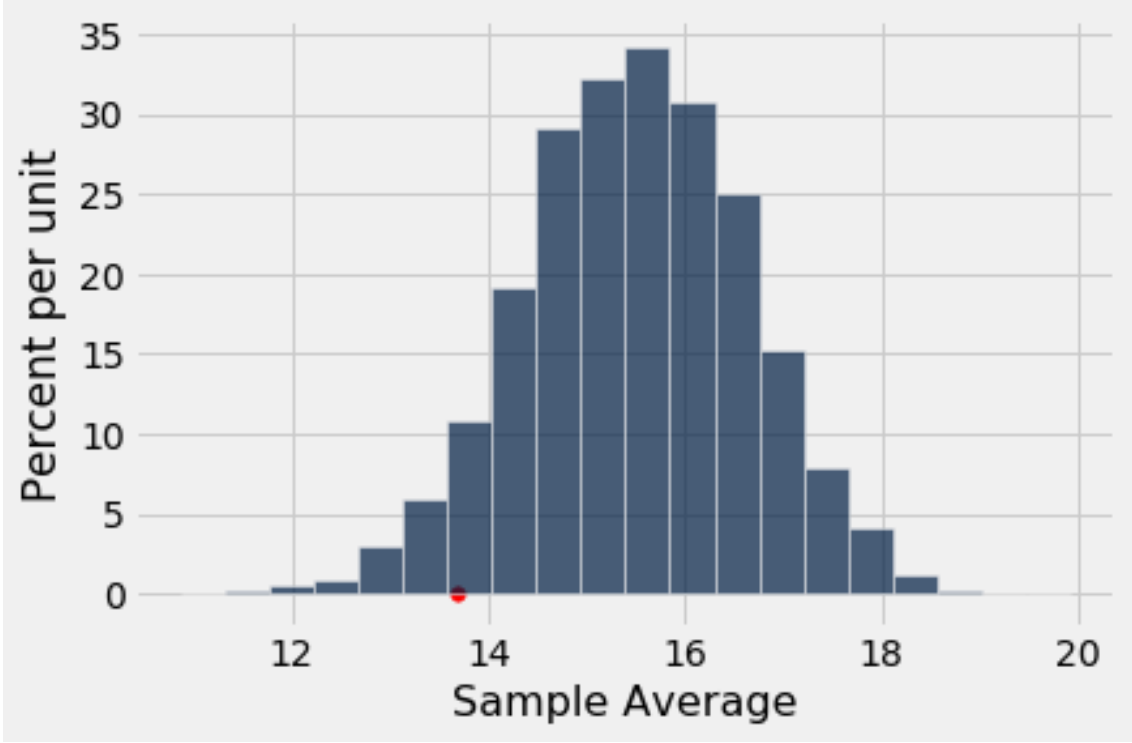
P(A) = (the number of times the sampled statistic was less than or equal to the observed statistic) divided by the number of samples



Compute the p-value

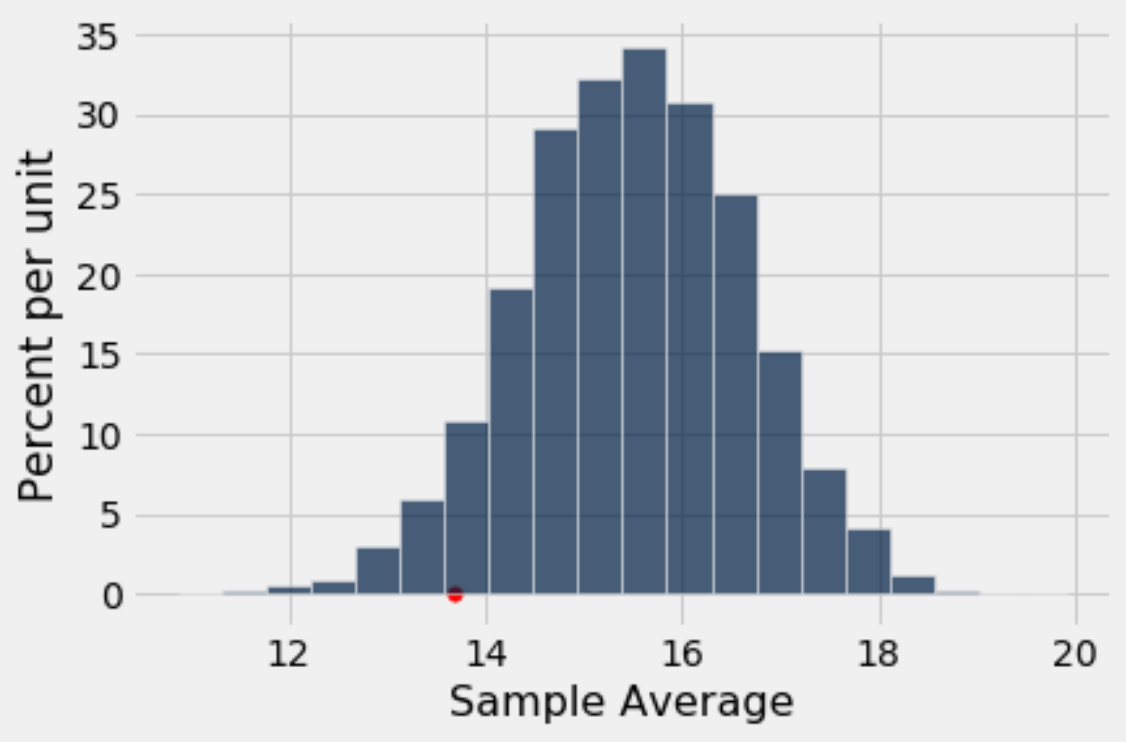
P(A) =

$$\frac{sum(sample\ averages \leq observed\ averages)}{50K}$$

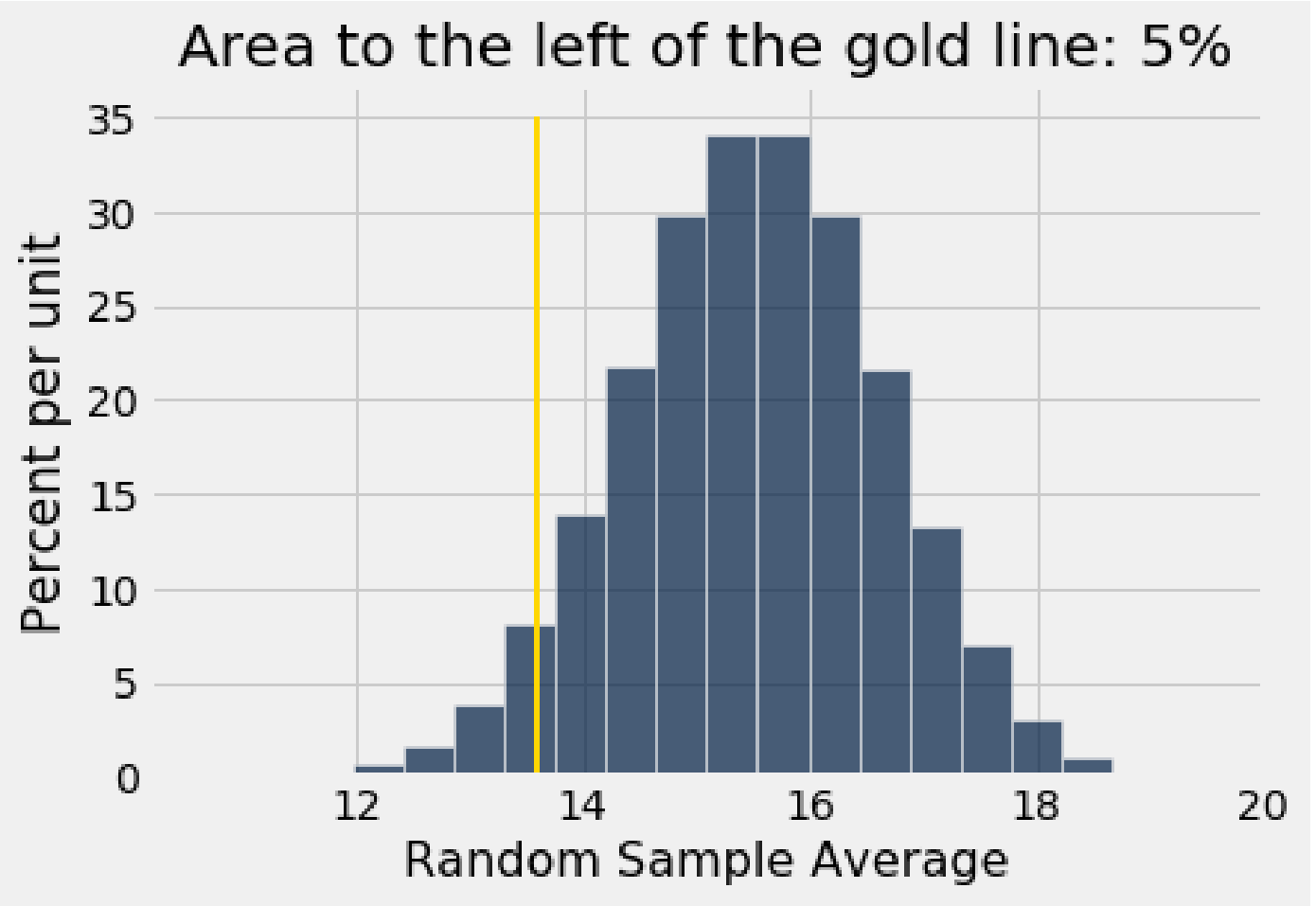


Compute the p-value

$$P(A) = 0.05682 \approx 5\%$$



Compute the p-value



Comparing Two Samples:

A/B Testing

Terminology

Compare values of sampled *individuals* in **Group A** with values of sampled *individuals* in **Group B**.

Question: Do the two sets of values come from the same underlying distribution?

Answering this question by performing a statistical test is called **A/B testing**.

The Groups and the Questions

Random sample of mothers of newborns. Compare:

A. Birth weights of babies of mothers who smoked during pregnancy
B. Birth weights of babies of mothers who didn't smoke

Question: Could the difference be due to chance alone?

Hypotheses

Null Hypothesis:

- In the population, the distributions of the birth weights of the babies in the two groups are the same. (They are different in the sample just due to chance.)

Alternative Hypothesis:

- In the population, the babies of the mothers who smoked weigh less, on average, than the babies of the non-smokers

Test Statistic

Group A: non-smokers
Group B: smokers

Statistic:
Difference between average weights:

- Group B average - Group A average

Negative values of this statistic favor the alternative

Simulating Under the Null

If the null is true, all rearrangements of labels are equally likely

Permutation Test:
Shuffle all birth weights
Assign some to Group A and the rest to Group B

- Key: keep the sizes of Group A and Group B that same from before

Find the difference between the two shuffled groups
Repeat

Random Permutations

tbl.sample(n)

Table of n rows picked randomly with replacement

tbl.sample()

- Table with same number of rows as original **tbl**, picked randomly with replacement

tbl.sample(n, with_replacement = False)

- Table of n rows picked randomly without replacement

tbl.sample(with_replacement = False)

- All rows of tbl, in random order


Types of Tests
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Hypothesis Testing Review

1 Sample: One Category *(e.g. percent of black male jurors)*
Test Statistic: empirical_percent, abs(empirical_percent - null_percent)
How to Simulate: sample_proportions(n, null_dist)

1 Sample: Multiple Categories *(e.g. ethnicity distribution of jury panel)*
Test Statistic: tvd(empirical_dist, null_dist)
How to Simulate: sample_proportions(n, null_dist)

1 Sample: Numerical Data *(e.g. scores in a lab section)*
Test Statistic: empirical_mean, abs(empirical_mean - null_mean)
How to Simulate: population_data.sample(n, with_replacement=False)

2 Samples: Numerical Data *(e.g. birth weights of smokers vs. non-smokers)*
Test Statistic: group_a_mean - group_b_mean,

- group_b_mean - group_a_mean, abs(group_a_mean - group_b_mean)

How to Simulate: empirical_data.sample(with_replacement=False)


Causality
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Randomized Controlled Experiment

Sample A: **control group**
Sample B: **treatment group**

if the treatment and control groups are selected at random, then you can make causal conclusions.

Any difference in outcomes between the two groups could be due to

- chance
- the treatment

Randomized Assignment & Shuffling