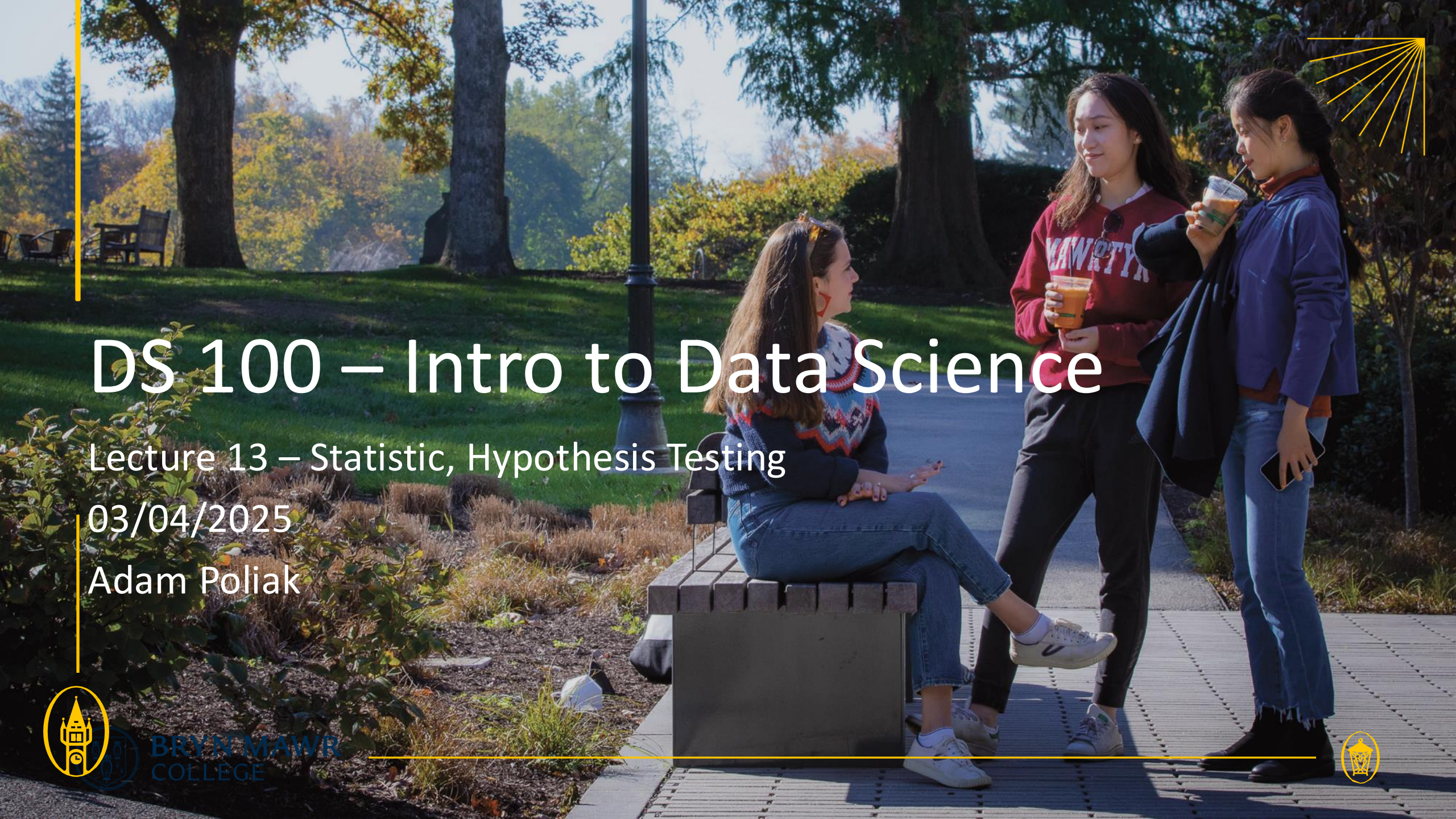


DS 100 – Intro to Data Science

Lecture 13 – Statistic, Hypothesis Testing

03/04/2025

Adam Poliak

BRYN MAWR COLLEGE

Announcements

HW05 – Probability, Simulation, Estimation, and Assessing Models

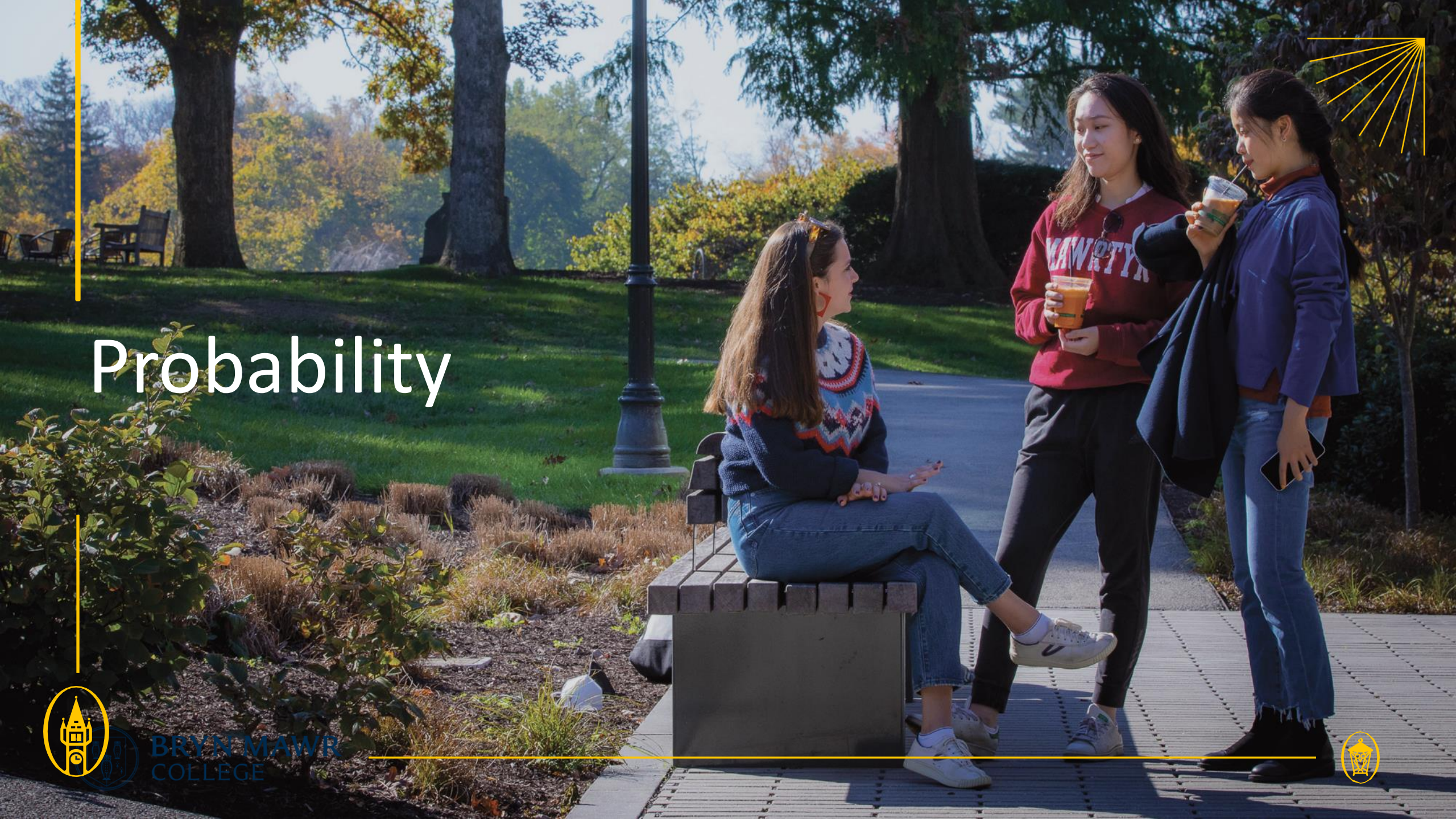
- Due Wednesday (03/05)

Lab05 – Examining the Therapuetic Touch

- Due Friday (03/07)

HW06 – Hypothesis Testing

- Due Wednesday (03/19)


Probability
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Basics

Lowest value: 0

- Chance of event that is impossible

Highest value: 1 (or 100%)

- Chance of event that is certain

If an event has chance 70%, then the chance that it doesn't happen is:

- 100% - 70% = 30%
- 1 – 0.7 = 0.3
- We call this the **Complement**

Complement: be careful

A = the event of sampling (with replacement) 5 aces in a row from a deck of card. P(A) = ?

- $\frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52}. = \frac{1}{52}^{5}$

What is the complement of A?

1. Drawing 5 cards and never getting an ace
2. Drawing 5 cards and not getting 5 aces

Complement: be careful

B = the event of sampling (with replacement) 5 cards and never getting an ace. P(B) = ?

- $\frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52} \times \frac{48}{52}. = \frac{48^5}{52}$

P(A) = $\frac{1}{52}^5$; P(B) = $\frac{48^5}{52}$

Is P(A) = 1 – P(B)?

- P(A) = $\frac{1}{52}^5 \cong \frac{1}{380M}$
- P(B) = $\frac{48^5}{52} \cong \frac{254M}{380M}$

Complement: be careful

A = the event of sampling (with replacement) 5 aces in a row from a deck of card. P(A) = ?

- $\frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52} \times \frac{1}{52}. = \frac{1}{52}^{5}$

The complement of A is:

1. P(Drawing 5 cards and never getting an ace) = $\frac{254M}{380M}$
2. P(not A) = $1 - \frac{1}{52}^{5} \cong \frac{380M - 1}{380M}$

Probability & Sampling

Distributions

Large Random Samples


A Statistic
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Why bother sampling?

Probability

Statistics

Sampling

Inference

Statistical Inference:

- Making conclusions based on data in random samples

fixed

Example:

- Use the data to guess the value of an unknown number

Depends on the random sample

- Create an **estimate** of an unknown quantity

Terminology

Parameter

- Numerical quantity associated with the population

Statistic

- A number calculated from the sample

A statistic can be used as an **estimator** of a parameter

Probability distribution of a statistic

Values of a statistic vary because random samples vary

"Sampling distribution" or "probability distribution" of the statistic:

- All possible values of a statistic
- and all corresponding probabilities for each possible values

Can be hard to calculate:

- Either have to do math
- Or generate all possible samples and calculate the statistic based on the each sample

Empirical Distribution of a Statistic

Based on simulated values of a statistic

Consists of all observed values of the statistic,
and the proportion of times each value appeared

Good approximation to the probability distribution of a statistic

- If the number of repetitions in the simulation is large

Hypothesis Testing

Choosing Between Two Viewpoints

Based on data:

- “Chocolate has no effect on cardiac disease”
- “Yes, it does”

Questions that we will consider:

- Were data was drawn?
- How the data was drawn?
- What can we conclude from the data?

Assessing models

Models

A model is a set of assumptions about the data

In data science, many models involve assumptions about processes that involve randomness:

- "Chance models"

Key question: does the model fit the data?

Approach to Assessing Models

If we can simulate data according to the assumptions of the model, we can learn what the model predicts

We can compare the model's predictions to the observed data

If the data and the model's predictions are not consistent, that is evidence against the model

Jury Selection

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Swain vs Alabama, 1965

Talladega County, Alabama

Robert Swain, black man convicted of crime

Appeal: one factor was all white-jury

Only men 21 years or older were allowed to serve

26% of this population were black

Swain’s jury panel consisted of 100 men

8 men on the panel were black

Supreme Court Ruling

About disparities between the percentages in the eligible population and the jury panel, the Supreme Court wrote:

- "… the overall percentage disparity has been small and reflects no studied attempt to include or exclude a specified number of Negros"

Supreme Court denied Robert Swain's appeal

Supreme Court Ruling in Data

Paraphrase: 8/100 is less than 26%, but not different enough to show Black men were systematically excluded

Question: is 8/100 a realistic outcome if the jury panel selection process were truly unbiased?

Sampling from a Distribution

Sample at random from a categorical distribution

```
sample_proportions(sample_size, pop_distribution)
```

Samples at random from the population

- Returns an array containing the distribution of the categories in the sample

Steps in Assessing a Model

Choose a statistic that will help you decide whether the data support the model or an alternative view of the world

Simulate statistic under the assumptions of the model

Draw a histogram of the simulated values

- This is the model's prediction for how the statistic should come out

Compute the statistic from the sample in the study

- If the two are not consistent => evidence against the model
- If the two are consistent => data supports the model ***so far***


A Genetic Model
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Mendel's genetic model

Pea plants of a particular kind
Each one has either purple flowers or white flowers

Mendel's model:

- Each plant is purple-flowering with chance 75%, regardless of the colors of the other plants

Question:

- Is the model good or not?

Choose a statistic

Take a sample, see what percent are purple-flowering

If that percent is much larger or much smaller than 75, that is evidence against the model

Distance from 75 is key

Statistic:

- | sample percent of purple-flowering plants – 75 |

If the statistic is large, that is evidence against the model

Model and Alternative

Jury Selection:

- **Model:** The people on the jury panels were selected at random from the eligible population
- **Alternative viewpoint**: No, they weren't

Genetics:

- **Model:** Each plant has a 75% chance of having purple flowers
- **Alternative viewpoint:** No, it doesnt

Steps in Assessing a Model

Choose a statistic to measure the “discrepancy” between model and data

Simulate the statistic under the model’s assumptions

Compare the data to the model’s predictions:

- Draw a histogram of simulated values of the statistic
- Compute the observed statistic from the real sample

If the observed statistic is far from the histogram, that is evidence against the model