

#### **Announcements**

#### **Checkpoint/Project 1:**

- Paired assignment that covers the previous section of the course material
- Due Friday 02/28

HW05 – Probability, Simulation, Estimation, and Assessing Models

Due Wednesday (03/05)





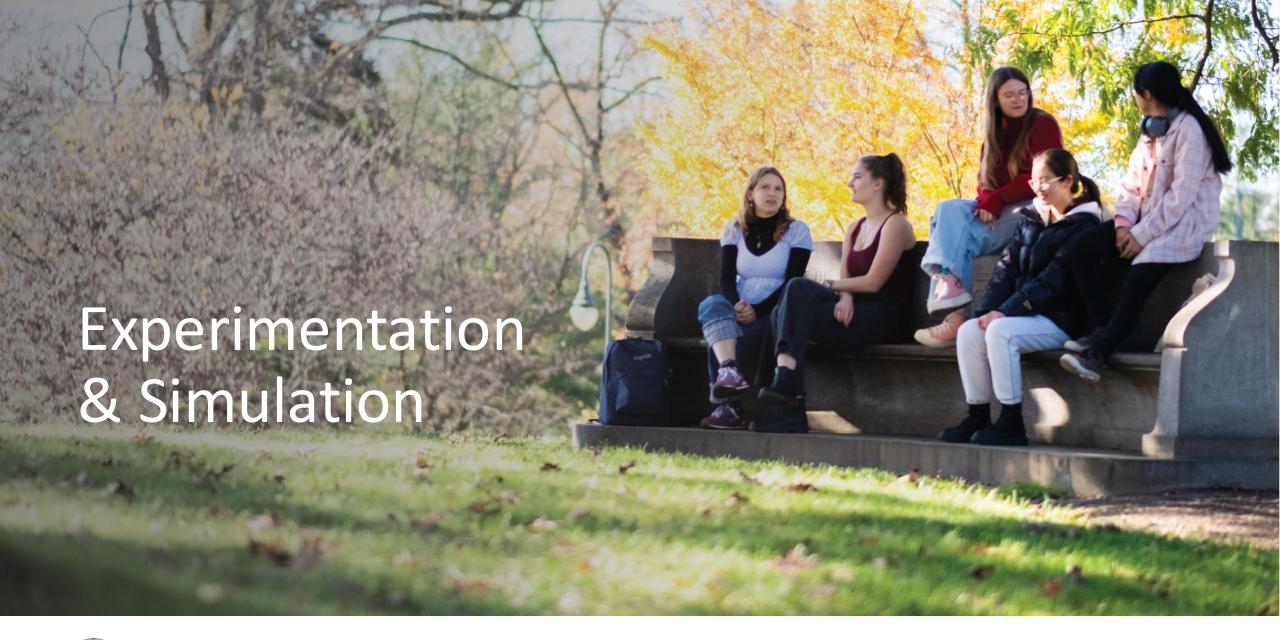
### Lab & Late Days

Can't use late days for lab

Lab 0-4, if you missed any, let me know (by end of today) and you can submit them with two late days









#### Experimentation

Why do we want to run experiments?

To test a hypothesis

What could a hypothesis be in our previous (coin flipping) experiments? Is a coin fair or not?

• • •





- Step 1: Choose a measurement/statistic to study Textbook lingo: what to simulate
- Step 2: Figure out how to compute the measurement Textbook lingo: figure how out to simulate the statistic
- Step 3: Choose how many times to simulate the statistic Textbook lingo: Number of Repetitions
- Step 4: Do it!

  Textbook lingo: simulate multiple values





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#### How to simulate multiple values

```
Collection array empty array to store the simulated values/statistic make_array()
```

Create a "repetitions sequence"

A sequence as long as the number of iterations
For *n* repetitions, use the sequence np. arange(n)

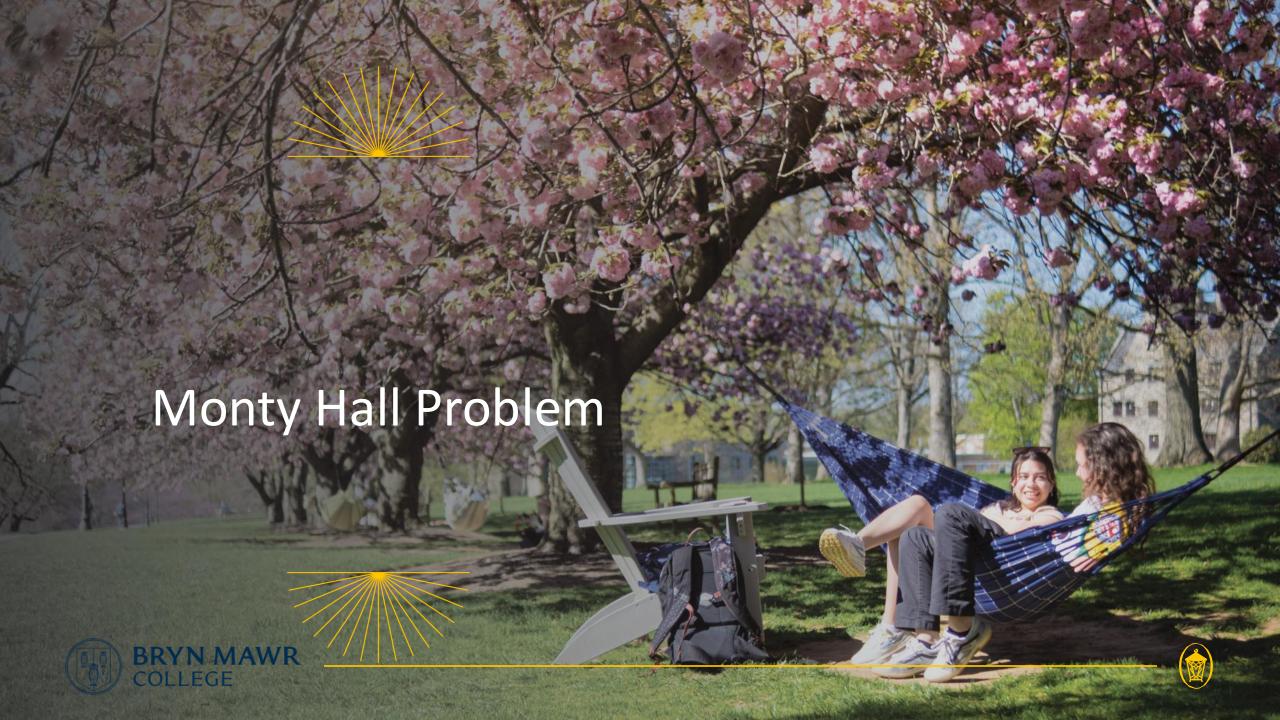
Create a for loop. For each element:

Simulate *one* value by using the function you wrote in Step 2

Augment the collection array with this simulated value





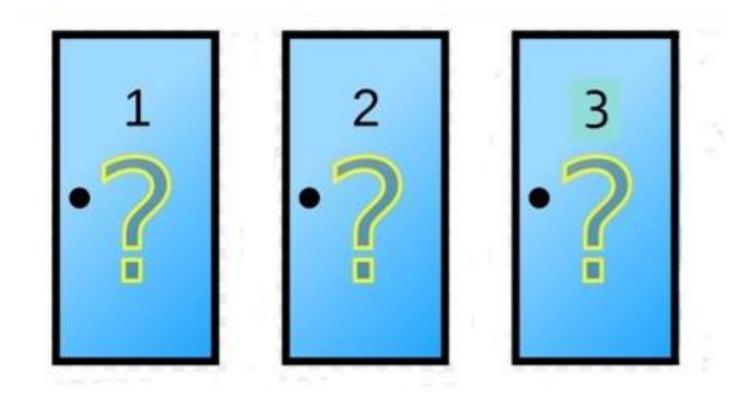








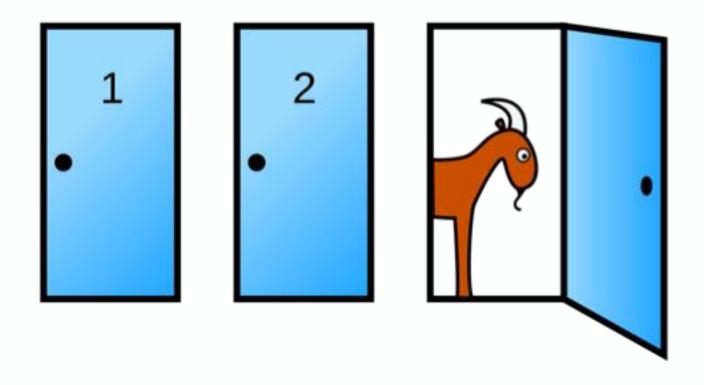
# Monty Hall Problem





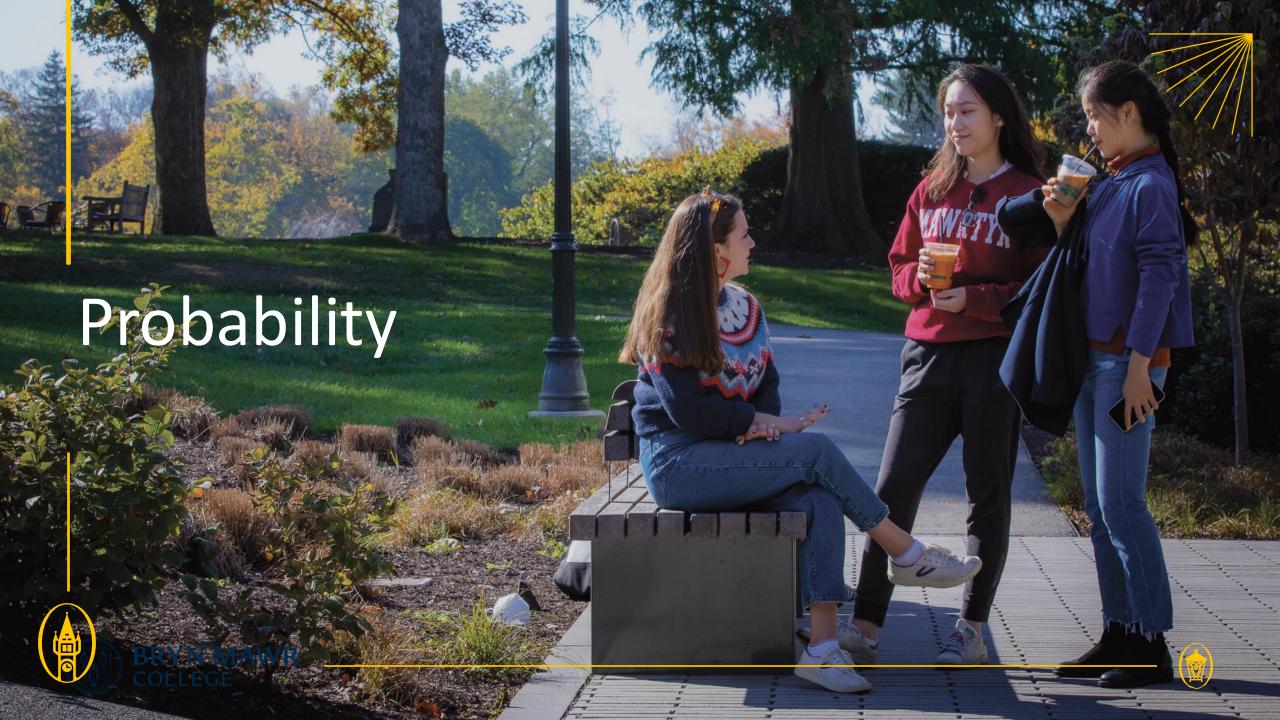


# Monty Hall Problem









#### Basics

Lowest value: 0

Chance of event that is impossible

Highest value: 1 (or 100%)

Chance of event that is certain

If an event has chance 70%, then the chance that it doesn't happen is:

- 100% 70% = 30%
- 1 0.7 = 0.3
- We call this the Complement





### **Equally Likely Outcomes**

**Assuming** all outcomes are equally likely, the chance of an event A is:

$$P(A) =$$

number of outcomes that make A happen total number of outcomes





#### Question

I have 3 cards: ace of hearts, king of diamonds, and queen of spades

I shuffle them and draw two cards at random without replacement.

What is the chance that I get the Queen followed by the King?





### Approach 1: Enumerate all outcomes

What is the chance that I get the Queen followed by the King?

- 1.Queen, King
- 2.Queen, Ace
- 3.Ace, King
- 4.Ace, Queen
- 5.King, Queen
- 6.King, Ace





### Approach 1: Enumerate all outcomes

What is the chance that I get the Queen followed by the King?

- 1. Queen, King
- 2.Queen, Ace
- 3.Ace, King
- 4.Ace, Queen
- 5.King, Queen
- 6.King, Ace





### Approach 1: Enumerate all outcomes

What is the chance that I get the Queen followed by the King?

- 1. Queen, King
- 2.Queen, Ace
- 3.Ace, King
- 4.Ace, Queen
- 5.King, Queen
- 6.King, Ace

Answer: 1/6





#### Approach 2: Probabilities of the sequences

What is the chance that I get the Queen followed by the King?

What's the probability I first draw Queen and what's the probability I then draw King





### Approach 2: Probabilities of the sequences

#### Step 1:

- Draw Queen from {Ace, King, Queen}
- What's the probability of drawing Queen?

1/3

#### Step 2:

- Draw King from {King, Ace}
- What's the probability of drawing King?

1/2

#### Combining them:

• What's 1/2 of 1/3?

1/6





### Multiplication Rule

Chance that two events A and B both happen

- = P(A happens) x P(B happens given that A has happened)
  - The answer is less than or equal to each of the two chances being multiplied
  - The more conditions you have to satisfy, the less likely you are to satisfy them all





#### **Addition Rule**

If event A can happen in exactly one of two ways, then

$$P(A) = P(first way) + P(second way)$$

The answer is greater than or equal to the chance of each individual way





# Complement: At least one head

What the probability that I flip coins and I get at least one head?

#### In 3 tosses:

- Any outcome except TTT (tails, tails)
- $P(TTT) = (1/2) \times (1/2) \times (1/2) = 1/8$
- P(at least one head) = 1 P(TTT) = 1 (1/8) = 87.5%

#### In 10 tosses:

•  $1 - (1/2)**10 \cong 99.9\%$ 



